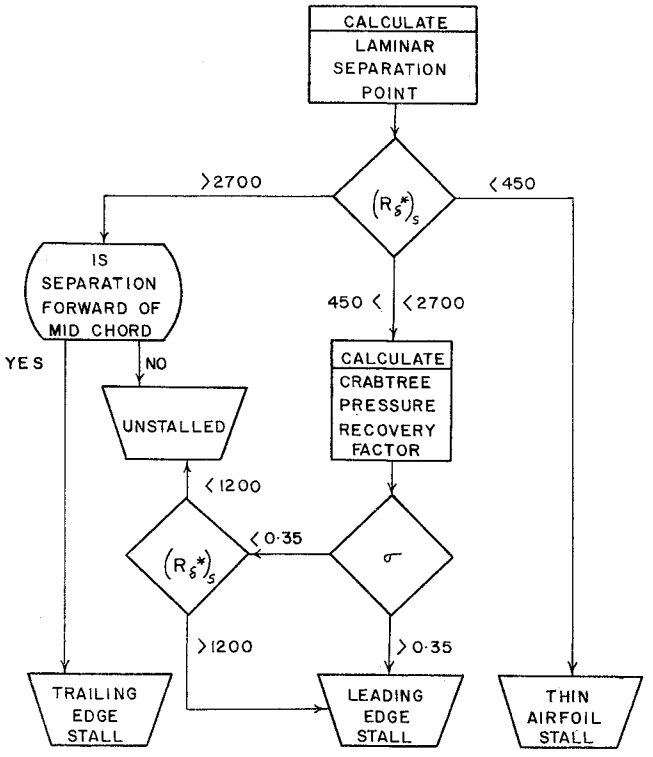

Fig. 2 Stall prediction scheme.

when the maximum shear stress that can be tolerated by the reattaching turbulent boundary layer is exceeded and has derived a pressure recovery factor.

$$\sigma = \frac{(C_p)_{\text{transition}} - (C_p)_{\text{separation}}}{1 - (C_p)_{\text{separation}}}$$

to represent this stress. Experimental investigations have determined $\sigma = 0.35$ to be the limiting value above which the bubble bursts suddenly and the flow breaks down into the pattern associated with fully separated flows. An alternate cause of short-bubble breakdown is a turbulent separation immediately downstream of reattachment. This is referred to as reseparation and only occurs when $(R_{\delta^*})_s > 1200$.

Summary

Some of the mechanism of airfoil stall have been discussed and parameters easily measured experimentally or calculated by most numerical solutions of boundary-layer equations selected to predict stalling behavior. Figure 2 summarizes the procedures outlined in this note and should be easily incorporated into existing airfoil analysis methods.

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Vibration Analysis of Complex Structures Consisting of Many Members

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LINEAR elastic properties of complex aircraft and spacecraft structures consisting of many members are commonly represented by the stiffness influence coefficients matrix. In vibration analysis of such structures, this matrix is used for both the "lumped masses" and the "assumed modes" methods, although the latter may be adapted to continuous media. However, the stiffness influence coefficients matrix represents not only a numerical evaluation of complicated stress-strain integrals but also a very rigid procedure for discretizing continuum properties. It leads to very high order matrices if calculations are intended beyond the first few natural frequencies and mode shapes of a structure. Experience shows that the discrepancy between the necessary order of the matrices and the actually calculable number of eigenvalues increases sharply and the scope of work becomes excessive, even with advanced computer facilities. This may be attributed to the rapid degeneration of the stiffness influence coefficients matrix as its reference points converge, yielding to the stiffness influence function [see, e.g., Eq. (27), Ref. 2, which represents the divergent development for the stiffness influence function in series of eigenfunctions]. Thus, the stiffness influence coefficients matrix seems to be a barrier to more refined vibration calculations for continuous but complex systems, where also the well-known straightforward differential or integral equation methods fail due to inextricable boundary conditions or kernel functions, respectively.

In an effort to avoid both the difficulties of boundary conditions and kernel functions, and the uncertainties of higher-order stiffness influence coefficients matrices, a "direct method" procedure was developed to compute the generalized stiffness matrix directly from the design data of the structure as part of the assumed modes method.[1] A set of assumed modes $\Im_r\langle x,y,z\rangle$; $r,s = 1,2,3,\ldots,\mathbf{r}$ (where the number $\mathbf{r}$ is at most two or three times the number of eigenmodes sought) is chosen to satisfy the conditions of the Rayleigh-Ritz method and some essential conditions of the Galerkin application. This means that the modes are roughly the same as those commonly used, chosen from previous analysis of similar structures, from exact solutions of neighboring systems, etc., but completed more or less intuitively in details. With these modes the elements φ_{rs} of the generalized stiffness matrix are computed by formula, as shown in the example, where they are written out in three-dimensional Cartesian coordinates.

The assumed modes need not be given analytically. The numerical evaluation of the integrals is programmed for the computer, which finally delivers the complete matrix ϕ, and, as a sort of by-product, the generalized mass matrix M. Each generalized stiffness element requires approximately the same amount of computer time as one element of a stiffness influence coefficient matrix, but there is a relatively small number, $\mathbf{r}^2$, of these generalized stiffness elements to be computed. The generalized stiffness elements are built up of solely additive parts. Each part is computed separately and concerns a well-defined part of the strain energy of a structural element. In this way it is relatively easy to

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follow subsequent design modifications of an aircraft in its developmental stage, or to program local stiffness variations for optimizations. Modifications of any single generalized stiffness element do not affect other elements. This means that the order of the matrices ϕ and M can readily be modified without having to change other elements. This would not be the case if stiffness influence coefficients were used. Since the direct method is not based on a network of reference points, the structural elements can be subdivided as finely as needed (up to the theoretical point where the parts become a continuum) to attain any desired degree of accuracy, without causing the numerical evaluation process to degenerate. Coarser or finer subdivisions, as well as partial or complete modifications of any assumed mode, can be used to prove the adequacy of the chosen assumptions.

Programmed in FORTRAN for the IBM 7040 and IBM 360/40, "direct method" was applied to a complex structure consisting of a tapered low-aspect-ratio, swept-back, vertical fin of a high-speed aircraft. The method yielded specific results which, introduced into a set of natural vibration equations, agreed fairly well with ground resonance tests.

Example

Elements φ_{rs} of the generalized stiffness matrix:

$$\varphi_{rs} = \begin{cases} \text{for } s = 1,2,\ldots,\mathrm{r}_1 & \mathrm{r}_1+1,\ldots,\mathrm{r}_2 & \mathrm{r}_2+1,\ldots,\mathrm{r}_3 \\ \text{for } r = 1,2,\ldots,\mathrm{r}_1 & & \\ \int_V \left(\frac{\partial}{\partial x}\mathfrak{F}_{rx}h_{11}\frac{\partial}{\partial x}\mathfrak{F}_{sx} + \frac{\partial}{\partial y}\mathfrak{F}_{rx}h_{44}\frac{\partial}{\partial y}\mathfrak{F}_{sx} + \frac{\partial}{\partial z}\mathfrak{F}_{rx}h_{66}\frac{\partial}{\partial z}\mathfrak{F}_{sx}\right)dV & \int_V \left(\frac{\partial}{\partial x}\mathfrak{F}_{rx}h_{12}\frac{\partial}{\partial y}\mathfrak{F}_{sy} + \frac{\partial}{\partial y}\mathfrak{F}_{rx}h_{44}\frac{\partial}{\partial x}\mathfrak{F}_{sy}\right)dV & \int_V \left(\frac{\partial}{\partial x}\mathfrak{F}_{rx}h_{13}\frac{\partial}{\partial z}\mathfrak{F}_{sz} + \frac{\partial}{\partial z}\mathfrak{F}_{rx}h_{66}\frac{\partial}{\partial x}\mathfrak{F}_{sz}\right)dV \\ \text{for } \mathrm{r}_1+1,\ldots,\mathrm{r}_2 & & \\ \int_V \left(\frac{\partial}{\partial y}\mathfrak{F}_{ry}h_{21}\frac{\partial}{\partial x}\mathfrak{F}_{sx} + \frac{\partial}{\partial x}\mathfrak{F}_{ry}h_{44}\frac{\partial}{\partial y}\mathfrak{F}_{sx}\right)dV & \int_V \left(\frac{\partial}{\partial y}\mathfrak{F}_{ry}h_{22}\frac{\partial}{\partial y}\mathfrak{F}_{sy} + \frac{\partial}{\partial x}\mathfrak{F}_{ry}h_{44}\frac{\partial}{\partial x}\mathfrak{F}_{sy} + \frac{\partial}{\partial z}\mathfrak{F}_{ry}h_{55}\frac{\partial}{\partial z}\mathfrak{F}_{sy}\right)dV & \int_V \left(\frac{\partial}{\partial y}\mathfrak{F}_{ry}h_{23}\frac{\partial}{\partial z}\mathfrak{F}_{sz} + \frac{\partial}{\partial x}\mathfrak{F}_{ry}h_{55}\frac{\partial}{\partial y}\mathfrak{F}_{sz}\right)dV \\ \text{for } \mathrm{r}_2+1,\ldots,\mathrm{r}_3 & & \\ \int_V \left(\frac{\partial}{\partial z}\mathfrak{F}_{rz}h_{31}\frac{\partial}{\partial x}\mathfrak{F}_{sx} + \frac{\partial}{\partial x}\mathfrak{F}_{rz}h_{66}\frac{\partial}{\partial z}\mathfrak{F}_{sx}\right)dV & \int_V \left(\frac{\partial}{\partial z}\mathfrak{F}_{rz}h_{32}\frac{\partial}{\partial y}\mathfrak{F}_{sy} + \frac{\partial}{\partial y}\mathfrak{F}_{rz}h_{55}\frac{\partial}{\partial z}\mathfrak{F}_{sy}\right)dV & \int_V \left(\frac{\partial}{\partial z}\mathfrak{F}_{rz}h_{33}\frac{\partial}{\partial z}\mathfrak{F}_{sz} + \frac{\partial}{\partial y}\mathfrak{F}_{rz}h_{55}\frac{\partial}{\partial y}\mathfrak{F}_{sz} + \frac{\partial}{\partial x}\mathfrak{F}_{rz}h_{66}\frac{\partial}{\partial x}\mathfrak{F}_{sz}\right)dV \end{cases}$$

with the constants (E = Young's modulus, ν = Poisson's ratio)

$$h_{11} = h_{22} = h_{33} = \frac{1-\nu}{(1+\nu)(1-2\nu)}\cdot E$$

$$h_{12} = h_{21} = h_{13} = h_{31} = h_{23} = h_{32} = \frac{\nu}{(1+\nu)(1-2\nu)}\cdot E$$

$$h_{44} = h_{55} = h_{66} = \frac{1}{2(1+\nu)}\cdot E$$

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